Degradation in Steels: Transformation Plasticity

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ABSTRACT

Super-plastic deformation that originates from transformation plasticity has favorable aspects for steels with improved strength and ductility. However, it also causes undesirable deformation of products or specimens, leading to their degradation. This article reviews recent investigations of transformation plasticity. A combination of newly suggested models, numerical analyses, and novel experiments has attempted to reveal the mechanism. Since the nature of the transformation plasticity is still unclear, there are significant challenges still to be solved. Fundamental understanding of transformation plasticity will be essential for the development of advanced steels.

Key words : Transformation plasticity, Degradation, Steel, Phase transformation, Creep

1. Background

Transformation plasticity has been believed to be a mechanism that causes permanent deformation during phase transformations of polycrystalline materials even under their yield stresses. This deformation has two sides from an engineering aspect, one is the improvement of product qualities, the other is degradation. The improvement means a usage of the transformation plasticity in the contexts of strength and ductility.^{1,2)} Steels made for these purposes, so-called transformation-induced plasticity (TRIP) steels,³⁾ have attracted great interest in the automotive industry. Degradation means that the deformation due to transformation plasticity is undesirable for products or specimens, and that it therefore degrades material qualities. For example, the situations listed below describe degradation in processing or experiments.

(a) Hot-rolled coils sometimes contract asymmetrically during steel making/storing processes. Transformation plasticity is believed to be the primary reason.⁴⁾

(b) Permanent deformations and residual stresses occur in welding processes.⁵⁾ Considerable portions of deformation might be caused by transformation plasticity.

(c) Dilatation curves that contain the contribution of transformation plastic deformation may cause significant errors in dilatometry.^{6,7)}

(d) Pop-ins are usually observed in nano-indentation on steels. Some of these are believed to originate from strain-induced displacive transformation.⁸⁾

Since super-plastic deformation due to transformation plasticity was reported, many researchers have begun to investigate its origin and nature extensively. Many theories and models have been proposed, which have been categorized into two main groups. For diffusive transformation, Greenwood and Johnson⁹⁾ derived an analytical solution for transformation plastic deformation on ideally plastic materials. They assumed that plastic deformation occurs in a weaker phase to accommodate external and internal stresses by volume mismatches between two solid phases. This model has been modified, extended and reproduced by other researchers. For example, Leblond et al. demonstrated the interaction between classical plasticity and TRIP,^{10,11)} Taleb and coauthors re-evaluated the Leblond model by providing various experimental grounds,¹²⁻¹⁴⁾ Fischer et al. quantified the effect of the orientation on the deformation of shape memory alloys;¹⁵⁻¹⁷⁾ Mahnken et al. utilized a unit-cell RVE model to find the mechanical behavior of macroscopic austenite/martensite composite, which combines the effect of classical plasticity and transformation induced plasticity.¹⁸⁾ On the other hand, Magee paid attention to the displacive martensitic transformation by which transformation plastic deformation is also induced.¹⁹⁾ The preference of a specific variant under a certain stress field was adopted to describe the deformation. Based on this theory, several models were proposed with extended or more generalized constitutive relations. Han et al.¹⁾ proposed a nucleation-controlled kinetics based on Olson and Cohen's approach^{3,20)} in order to compensate for the disadvantage of kinetics in the strain-induced martensitic transformation.²¹⁻²⁴⁾ Other approaches frequently adopted for the representation of the macroscopic behavior of phase transformed materials are used to introduce the micromechanical or multi-scale modeling techniques.^{15,25-32)} Recently, Levitas and Ozsoy developed a micromechanical approach by modeling the universal ther-

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modynamic driving force for the interface reorientation to derive the stress- or strain-induced martensite transformation. In the application part of their micromechanical model, various types of representative volumes were built to validate the effect of the athermal threshold, the martensite variants and an interface orientation under three dimensional thermo-mechanical loading.^{28,29,33)}

In this article, recent investigations of transformation plasticity, which attempted to reveal its mechanism and nature, are reviewed. These investigations include newly developed theories, numerical approaches, and novel experiments.

2. Representative theories: newly suggested models

2.1. Accelerated creep model³⁴⁾

During the diffusive phase transformation, the phase interface migrates through the movement of atoms across the interface. Generally, it can be assumed that the overall atomic flux across the interface will be perpendicular to the interface and that the migrating atoms will rearrange at the nearest atomic site in the transformed phase. However, when an external stress is applied, the migrating atoms will move to positions where they can release the applied stress field, and this can give rise to an atomic flux along the phase interface. This phenomenon may be similar to the mechanism of Coble creep.

The model of Greenwood and Johnson has been modified to include the temperature-dependence of the strain rate caused by the transformation plasticity.^{35,36)} These models could simulate the thermally activated behavior of the transformation plasticity in a relatively high temperature range. All of these models were based on the volume mismatch between the two phases and the creep deformation of the weaker phase during the phase transformation under externally applied stress. Recently, however, several sets of experimental results have been reported, which were difficult to explain by the Greenwood and Johnson's consideration. One of these is that an externally applied stress, even much lower than the yield stress of the material, induces considerable permanent strain during the recrystallization and growth in extra low-carbon steels.³⁷⁾ Since there is little volume mismatch between the unrecrystallized and recrystallized region, the evolution of permanent strain during recrystallization is difficult to interpret by the volume mismatch induced internal stress model. Besides, it was found that the deviation angle from the Kurdjumov-Sachs (KS) orientation relationship considerably increased with the uniaxially applied stress during the austenite-to-ferrite transformation of steel,^{38,39)} which implies that the sliding of the migrating transformation interface possibly occurs when the external stress is applied during phase transformation. From the above experimental observations, the transformation plasticity behavior of steel during the phase transformation under externally applied stress is modeled on the basis of a migrating interface diffusion mechanism, which is described as an accelerated Coble creep. The mathematical form of the model can be expressed as follows:

$$\dot{\varepsilon} = \frac{1}{3} \delta_0 \dot{X} \frac{\sigma \Omega}{k_B T} c_{v0} \exp\left(-\frac{Q_f}{k_B T}\right) + \frac{B_c \delta D_I^s \sigma \Omega}{\pi d_0^3 k_B T} \quad (1)$$

where d_0 , δ , and Ω represent the initial grain size of the parent phase, the effective thickness of the interface and the volume of the vacancy, respectively. The Boltzmann constant, k_B , has a value of $1.38 \times 10^{-23} \text{ J/K}$. c_{v0} is a dimensionless constant determined by the change in thermal entropy associated with the formation of the vacancy and Q_f is the formation enthalpy of the vacancy at the interface. σ is the applied stress, and the X with a dot above it is the transformation rate. δD_I^s denotes the effective diffusion coefficient at the stationary interface. B_c is known to be 148 for Coble creep.⁴⁰⁾ The first term on the right side of Equation (1) indicates the component of the creep strain rate during the interface migration, which can be attributed to the progress of the phase transformation. When no phase transformation occurs, the first term on the right side of Equation (1) becomes zero and Equation (1) becomes the typical equation for Coble creep.

Fig. 1 shows the relationship between the transformation

Fig. 1. Transformation shape strain after transformation as a function of applied stress during (a) cooling and (b) heating by 2°C/sec .³⁴⁾

shape strain after the phase transformation and the applied stress. The transformation shape strain is proportional to the applied stress. This linear relationship is in agreement with the many experimental observations^{9,21,41,42)} of the transformation plasticity.

2.2. Mechanically-induced martensitic transformation¹⁾

A one-dimensional model for the kinetics of strain-induced martensitic transformation was developed by Olson and Cohen.²⁰⁾ Stringfellow et al. derived a constitutive model for the kinetics of mechanically-induced martensitic transformation that considered the external stress state in terms of the hydrostatic pressure and the equivalent shear stress.²¹⁾ Tomita et al. modified Stringfellow's model to account for the experimental finding that the deformation behavior is controlled by the shear band mode as the strain rate increases.²²⁻²⁴⁾ However, the previous models were derived only for the kinetics of strain-induced martensitic transformation and have a disadvantage in that the athermal martensitic transformation cannot be analyzed. In addition, the models dealt with the TRIP strain as a strain softening without respect to the microstructural change, which is caused by the preferential selection of variants under the external stress state.

Martensitic transformation kinetics was assumed as a nucleation-controlled phenomenon on the basis of Olson and Cohen's approach.^{20,21)} The probability that a nucleation site would really act was derived for each variant as a function of the interaction energy between the externally applied stress state and the lattice deformation based on the Kurdjumov-Sachs (K-S) relationship. The TRIP strain was also evaluated by assessing the difference of the nucleation rate of 24 K-S variants. Each of the 24 variants of the K-S relation has one compressive axis and two tensile axes for the martensitic transformation, as can be seen in Fig. 2; this is called the Bain distortion. To reflect the athermal martensitic transformation kinetics as well, Koistinen and Marburger's empirical formula⁴³⁾ was included in the probability equation.

Several predictions compared with the experimentals were provided, including the shift of M_s temperature (Fig. 3). In the case of both uniaxial tension and compression, it can be seen that M_s temperature is linearly raised by the increase

Fig. 2. Lattice correspondence between FCC and BCT and the Bain deformation accompanying the transformation.¹⁾

Fig. 3. Change of M_s temperature as a function of external stress under various stress conditions. Lines and symbols indicate the calculated and measured⁴⁴⁾ values, respectively.¹⁾

in applied stress. The increase of M_s temperature by tensile stress is larger than that by compressive stress. The raising of M_s temperature even by uniaxial compression is explained on the basis of the existence of one compressive axis for the martensitic transformation in the Bain distortion, as can be seen in Fig. 2. Under hydrostatic pressure, M_s temperature is lowered with the increase in pressure. The process can be described as shown that the hydrostatic pressure suppresses the volume expansion due to the transformation. It was confirmed that the criterion derived from the present model of mechanically-induced martensitic transformation could well predict the shift of M_s temperature measured under various stress conditions.

3. Representative Numerical Works

3.1. Dilatometric non-isotropy⁶⁾

Generally, it is known that non-isotropic volume changes in dilatometry have also been observed during the phase transformation, even in steel with an isotropic microstructure.⁴⁵⁾ The contribution of the non-isotropic volume change to the dilatation data was recently quantified and this change was incorporated into a dilatometric analysis model.⁴⁵⁾ However, up to now, there have been no studies that have attempted to pinpoint the cause of the non-isotropic dilatation in specimens with isotropic microstructure or to predict the non-isotropic dilatation of this kind of specimen.

Fig. 4(a) and (b) show typical examples of the dilatation curves for an ultra-low carbon (0.0003 wt%C-1.0 wt%Mn, ULC) and low carbon (0.02 wt%C-1.0 wt%Mn, LC) steel during cyclic heat treatments up to 900 and 1000°C, respectively. In the case of the ultra-low carbon steel, since the steel underwent no transformation in the temperature range up to 900°C, only thermal expansion and contraction could be observed. On the other hand, in the case of the low carbon steel, length change during the phase transformation could be found. Note that a

Fig. 4. Measured dilatation curves for (a) ultra-low carbon steel (steel ULC) during a thermal cycle with 1°C/s and (b) low carbon steel (steel LC) during a thermal cycle with 10°C/s.⁶⁾

mismatch at the starting and ending points of the dilation curve was observed in the case of the low carbon steel, which implies that the volume change accompanying the phase transformation has non-isotropic characteristics. Fig. 5 shows the microstructures of the dilatometric specimen before and after the thermal cycling of this low carbon steel. This microstructure was not notably changed by the thermal cycling with the given cooling and heating rates of 10°C/sec, and no specific microstructural directionality, such as a microstructural band, which refers to the alternating layers of ferrite and second phase, was observed.

In order to describe this non-isotropic dilatometric behavior during the phase transformation in steel, the concept of transformation plasticity was used. The constitutive equation for the transformation plasticity was incorporated into a finite-element (FE) model (Fig. 6), which was adopted to describe the non-isotropic dilatometric behavior during the phase transformation in the steel without any specific microstructural directionality. An implicit numerical solution procedure to calculate the deformation during the dilatometric experiment

Fig. 5. Microstructures of steel LC (a) before and (b) after heat treatment up to 900°C at 10°C/s.⁶⁾

Fig. 6. FE meshes and boundary conditions for heat transfer in the dilatometric system.⁶⁾

was incorporated into the general purpose implicit FE program. Besides the thermo-elasto-plastic constitutive equations, the phase transformation kinetics was characterized by a Johnson-Mehl-Avrami-Kolmogorov (JMAK) type equation. For the FE formulation, the stress increment was defined as:

$$d\sigma = C^e : (d\varepsilon^T - d\varepsilon^v - d\varepsilon^{TP} - d\varepsilon^p) \quad (2)$$

Where C^e and $d\varepsilon^e$ are the elastic stiffness tensor and the elastic strain increment, respectively. $d\varepsilon^v$, $d\varepsilon^{TP}$ and $d\varepsilon^p$ are the volumetric strain increment due to the phase transformation and temperature change, the transformation plasticity strain increment associated with the phase transformation and the plastic strain increment, respectively.

In order to estimate precisely the interfacial heat transfer rate on the surface of the dilatometric specimen, including the induction heating, an inverse heat transfer technique was used. The dilatometric behaviors of both conventional low carbon and ultra-low carbon steels were simulated by using the FE model. To validate the suggested FE model, the measurement of the dilatation curves was carried out for these low carbon and ultra-low carbon steels and we compared the simulation results with the experimental data (Fig. 7). From the simulation results, the origin of the non-isotropic dilatation in the specimens was discussed.

The transformation plasticity was caused by the small amount of stress that naturally developed in the specimen during the dilatometric experiment. In the conventional low carbon steel, the stress in the specimen formed mainly due to the very small external force supplied to support it during the dilatometric experiment. As regards the ultra-low carbon steel, whose phase transformation occurs within an extraordinarily narrow temperature range, the inhomogeneous phase transformation due to the thermal gradient along the radial direction in the specimen was mainly responsible for the stress field in the specimen during the dilatometric experiment.

3.2. Microstructurally banded steel⁷⁾

Microstructural bands in steels are alternating layers of ferrite and pearlite. They originate from the alignment of segregated regions with substitutional elements during casting and sub-

Fig. 7. Comparison between calculated (lines) and measured (symbols) dilatation curves. Curves A and B represent the dilatometric curve calculated without and with consideration of transformation plasticity, respectively.⁽⁶⁾

sequent hot-rolling, which eventually produces layers with different microstructural constituents. A number of studies⁴⁶⁻⁴⁹⁾ have considered the correlation between the microstructural directionality and the dimensional non-isotropy, which is associated with non-isotropic volume changes during phase transformations.

The orientation dependent dilatometric behavior in microstructurally banded steel was simulated using the finite element analysis combining the thermal, elastic, and conventional plasticity as well as the transformation plasticity (Fig. 8). To examine the effect of transformation plasticity on the non-isotropic dilatations, numerical calculations were carried out in two different ways: (a) not considering the transformation plasticity so that only the elastic, conventional plastic, and volumetric deformations were taken into account, and (b) considering the transformation plasticity as well. Fig. 9 compares the calculated and measured dilatation curves of the RD and ND (indicated in the figure) specimens. In the calculation not considering the transformation plasticity (Fig. 9(a)), the change in length of the RD specimen during cooling was larger than that of the ND specimen, which is inconsistent with the measured results, which show a smaller length change in the RD specimen. In the case of taking the transformation plasticity into account (Fig. 9(b)), the calculated changes in length in both directions captured the experimentally observed dimensional non-isotropy well. The results suggest that transformation plasticity plays a major role in generating the characteristic dilatometric behavior, derived from dimensional non-isotropy during transformations.

3.3. Asymmetric contraction of hot-rolled coil⁴⁾

A hot-rolled steel strip is generally stocked in the form of a hollow cylindrical coil after the hot rolling process. The hot coil is cooled from 500-700°C to room temperature over a 4-5-day period under natural air cooling conditions.^{50,51)} In most hot strip rolling processes, the phase transformation of the steel is finished on the run-out table (ROT) before coiling,⁵²⁾ and the hot coil is normally cooled down but maintained in a cylindrical shape.⁵³⁾ However, asymmetric contraction occurs in an actual mill during cooling after the coiling of hot-rolled steel, which has significantly high hardenability due to its high content of carbon or other alloying elements and shows

Fig. 8. (a) Microstructure of the hot-rolled steel. (b and c) Finite-element mesh for the dilatometric specimens: (b) RD specimen showing the longitudinal direction parallel to the rolling direction, and (c) ND specimen showing the longitudinal direction perpendicular to the rolling direction.⁽⁷⁾

Fig. 9. Comparisons between the calculated and measured dilatation curves: (a) not considering transformation plasticity; (b) considering transformation plasticity.⁷⁾

incomplete phase transformation prior to coiling, as can be seen in Fig. 10. This asymmetric contraction behavior is closely related to the phase transformation that occurs after coiling, and cannot be described by conventional creep behavior. This shape change in the hot coil causes acute problems in industrial applications, such as serious scratching on the strip surface during uncoiling.

The constitutive equation for transformation plasticity was incorporated into a general purpose implicit FE program. In addition to the thermo-elasto-plastic constitutive equations, the phase transformation kinetics was characterized by a JMAK type equation. The validity of the proposed model was examined by reproducing the asymmetric contraction behavior of the coil. The effect of some selected process variables on the asymmetric contraction was investigated through a series of process simulations.

Considering the transformation plasticity, the asymmetric contraction behavior of the coil during cooling could be reproduced successfully using FE simulation (Fig. 11). It was confirmed that the asymmetric contraction was caused by the small stress that develops naturally in the hot-rolled coil due to gravity. The FE simulations showed that the extent of the phase transformation before coiling, the tension force during

Fig. 10. Example of asymmetric contraction of hot coil after coiling.⁴⁾

Fig. 11. Deformed shapes of the hot coil after cooling to room temperature; these shapes are calculated by one layer FE modeling without (a) and with (b) the consideration of the transformation plasticity, respectively. Displacement was enlarged by a factor of 1.5 for both cases.⁴⁾

coiling, and the steel weight per unit area of the inner layer in the coil are controllable process variables that can reduce the asymmetric contraction during cooling after the coiling of hot-rolled steel.

4. Representative Experimental Works

4.1. Variant selection in mechanically-induced martensitic transformation⁵⁴⁾

Variant selection in mechanically induced martensitic transformation of metastable austenite is investigated with respect to the interaction between external stress and lattice deformation of the transformation. The orientations of parent austenite and newly transformed martensite are measured for tensile and compressive deformation using electron back-scattered diffraction (EBSD). For an individual austenite grain, the orientation of the 24 K-S variants are evaluated and compared with the measured orientation of the martensite. The interaction energy between the externally applied stress and the lattice deformation is calculated for each 24 K-S variant and the probability of variant selection is assessed.

The assessed probability is compared with the experimental results (Fig. 12). The interaction energy was calculated (Fig. 13) from the Bain deformation and the invariant shear strain on the basis of the K-S orientation relationship. It was confirmed that the calculated interaction energy for the transformed martensitic variant has a relatively large negative value. The negative value of the interaction energy under the tensile and compressive stress state is related with the favorable interaction with the applied stress; this means that the probability that some specific variants will be selected increases due to the external stress.

4.2. Strain-induced martensitic transformation: nano-indentation⁹⁾

Experimental results of nanoindentation and microstructural studies were reported to provide micromechanical insight into the strain-induced phase transformation and deformation behavior of metastable austenite in TRIP steel. Sequential experiments were carried out, first using EBSD to map the phase and orientation distributions of the grains, followed by nanoindentation of individual austenite grains in the mapped region, then sectioning through an indent using focused ion beam (FIB) milling, and finally transmission electron microscopy (TEM) to confirm the formation of martensite from austenite under the indent. In addition, the load-displacement curves of the metastable austenite phase were analyzed to identify

Fig. 12. EBSD orientation mapping of (b) martensitic variants transformed from (a) single parent austenite grains after 10% tensile test. Martensites M1 and M2 seem to be transformed from austenite grains A1 and A2, respectively.⁵⁴⁾

signatures of the strain-induced martensitic transformation.

The load-displacement curve obtained from nanoindentation (Fig. 14) revealed two types of pop-in events on the loading segment. Based on a Hertzian analysis of the elastic portion

Fig. 13. The mechanical interaction energies calculated for 24 K-S variants of parent austenites (a) A1 and (b) A2 under the 10% tensile condition.⁵⁴⁾

Fig. 14. Comparison of experimental measurements (black symbols) to predictions of the Hertzian elastic load-displacement behavior (blue dashed line) of the metastable austenite grain. Arrows indicate the starting points of the three pop-ins.⁸⁾

Fig. 15. (a) Bright-field TEM image of the cross-section of a sample indented to a depth slightly beyond the third pop-in of Fig. 14. Diffraction patterns from (b) the region closest to the ferrite grain in (a) showing that it remains as austenite, γ , and (c) the region closest to the indenter in (a) showing that it has transformed to martensite, α' .⁸⁾

of the load-displacement curve, the first type was attributed to the elastic-to-plastic transition of austenite. Based on the mechanical interaction energy between the externally applied stress and the lattice deformation during nanoindentation, the second type of pop-in can be described as resulting from geometrical softening due to the selection of a favorable martensite variant. The existence of martensite after nanoindentation was confirmed by TEM analysis of the cross-section of an indented sample (Fig. 15).

5. Challenges and Further Studies

Although this article starts by looking at the undesirable aspects of transformation plasticity, the primary concern is not the degradation by itself, but rather its mechanism and nature. Fundamental understanding of transformation plasticity will be a key to future research. Many previously published studies have attempted to take advantage of transformation plasticity. The representative materials are TRIP aided steels,³⁾ which have engineering importance in the automotive industry. Twinning-induced plastic deformation is also attracting considerable interest for the possibility that it might lead to remarkable improvement in both strength and ductility. However, there is still a lack of fundamental investigations, possibly due to obstacles that derive from the requirements of nano-scale engineering. Transformation plasticity can hardly be observed or measured on the nano/micro-scale. Despite recent study that have adopted the phase field model (PFM) into the FE model in order to consider micro-scaled transformation plasticity,⁵⁵⁾ the mechanism and nature of transformation plasticity are not clear. Obviously, the requirements to understanding this uncertainty will increase, and many opportunities will arise. Future works will have to be carried

out as close collaborations between nano/micro-scale experiments and observations, along with numerical analysis.

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